

## ROTATION, DIFFUSION, AND OVERSHOOT IN THE SUN: EFFECTS ON THE OSCILLATION FREQUENCIES AND THE NEUTRINO FLUX

BRIAN CHABOYER

Canadian Institute for Theoretical Astrophysics, Toronto, Ontario, Canada, M5S 1A7

P. DEMARQUE

Center for Solar and Space Research, Department of Astronomy, Yale University, New Haven, CT 06520

D. B. GUENTHER

Department of Astronomy and Physics, Saint Mary's University, Halifax, Nova Scotia, Canada, B3H 3C3

AND

M. H. PINSONNEAULT

Department of Astronomy, Ohio State University, Columbus, OH 43210

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### ABSTRACT

We have studied the importance of the combined effects of rotation, diffusion, and convective overshoot on the  $p$ -mode oscillation spectrum and the neutrino flux of the standard solar model. To isolate the various physical effects included in the new rotation plus diffusion models we also constructed solar models to test the significance of diffusion and of overshoot by themselves. In previous studies, models that include helium diffusion during solar evolution were found to improve the predicted  $p$ -mode frequencies for some modes and worsen the agreement for others (Guenther et al. 1993). Here we verify this result for both the Bahcall & Loeb (1990) formulation of diffusion and the Proffitt & Michaud (1991) formulation of diffusion. We find that the effects of rotation on the Sun's structure in the outer layers perturbs the  $p$ -mode frequencies only slightly when compared to the more substantial effects due to diffusion. In the thin overshoot layer (taken here to be  $0.1 H_p$ ), we have compared the effect of overmixing in a radiative layer versus convective (adiabatic) penetration. Neither radiative overmixing nor adiabatic penetration has any significant effect on the  $p$ -modes, probably in part because the overshoot layer is constrained to be thin. The predicted neutrino flux in our diffusion plus rotation model is 7.12 SNU for Cl detectors, 127 SNU for Ga detectors, and  $5.00 \times 10^6$  ergs  $\text{cm}^{-2}$  for the  $^8\text{B}$  neutrinos; this is approximately half-way between the standard solar model without diffusion, and the standard solar model with diffusion alone.

*Subject headings:* convection — diffusion — elementary particles — Sun: interior — Sun: oscillations — Sun: rotation

### 1. INTRODUCTION

Standard solar models now predict a  $p$ -mode oscillation spectrum which agrees with the observed oscillation spectrum of the Sun (Guenther et al. 1992a, b; Guzik & Cox 1993), within the estimated uncertainties of the physical input. This is the result of many improvements in the input physics, most significantly the advances in astrophysical opacities (Kurucz 1991; Rogers & Iglesias 1994). The importance of diffusion processes, not normally included in the standard solar model calculation, have also been explored in solar models (Proffitt & Michaud 1991, hereafter PM; Guenther, Pinsonneault, & Bahcall 1993; Guzik & Cox 1993; Bahcall & Pinsonneault 1992a, b). The diffusion of helium from the surface convection zone into the radiative layers just below the convection zone was found to modify the predicted frequencies of  $p$ -modes that are most sensitive to the conditions near the base of the convection zone (Christensen-Dalsgaard, Proffitt, & Thompson 1993; Guenther et al. 1993; Guzik & Cox 1993). Specifically helium diffusion was found to bring the frequencies of  $p$ -modes with  $l = 30$ – $50$  into closer agreement with observations. Furthermore, the depth of the convection zone of the solar models that include diffusion is closer to the depth derived from inversion of the  $p$ -mode frequency observations (Christensen-Dals-

gaard, Gough, & Thompson 1991) than standard solar models that do not include diffusion.

The agreement is not perfect, though. Even better agreement was found when the efficiency of diffusion (Bahcall & Loeb 1990) was reduced by a factor of 2. In addition, it appears that the improvement in the  $p$ -modes with  $l$ -values from 30 to 50 is at the expense of other  $p$ -modes. That is to say, including diffusion affects adversely the agreement for modes that penetrate more deeply,  $l = 0$ – $30$ , and modes that do not penetrate to the base of the convection zone at all,  $l = 50$ – $100$  (Guenther et al. 1993).

In parallel with our work on diffusion in the Sun, we have studied the rotational history of the Sun following the approach of Endal & Sofia (1978, 1981). This work has been carried out using a version of the Yale code that includes some of the effects of rotation, such as angular momentum transfer and the associated chemical mixing due to rotational instabilities (Pinsonneault et al. 1989; Pinsonneault, Kawaler, & Demarque 1990). In this approach, the radial dependence of the angular velocity is specified in convective regions; in the fully convective pre-main-sequence phase, this plus the total angular momentum gives the initial conditions. Angular momentum is assumed to be lost from the surface convection

zone of the models due to a magnetic stellar wind. In radiative regions, angular momentum is conserved locally; the time-scales for angular momentum transport and mixing are then estimated and solved for, using a coupled set of diffusion equations. In such models, the spin-down of the outer convection zone creates a shear layer at its base.

Rotational mixing will counteract element separation; this is especially important below the convection zone where both mechanisms are the most effective. In addition, diffusion will interact with other mixing processes associated with the outward transfer of angular momentum from the interior of the Sun. This nonlinear interaction between rotationally induced mixing and diffusion has recently been modeled in a self-consistent way during solar evolution by Chaboyer (1993) and Chaboyer, Demarque, & Pinsonneault (1995a, hereafter CDP). Earlier work by Proffitt and Michaud had considered the competition between diffusion and a hypothetical "turbulent diffusion" law meant to simulate rotationally induced turbulence in the Sun.

The purpose of this paper is to further explore the effects of diffusion on the  $p$ -mode frequencies. We wish to determine if the effort to include a more realistic treatment of transport phenomena in solar models leads to better predictions of the solar  $p$ -mode oscillation frequencies. We also wish to investigate the importance and the nature of the thin transition layer at the base of the convection zone. In particular, we are interested in testing whether this transition overshoot layer is best represented by simple convective overmixing (in radiative equilibrium), or by convective penetration (in adiabatic equilibrium), as discussed by Zahn (1991, 1993).

Finally we estimate the effect of these improvements on the calculated neutrino flux. Bahcall & Pinsonneault (1992a, b) have computed solar models with the Yale code to evaluate the expected flux of solar neutrinos, and have explored the impact of helium diffusion and other changes in the input physics for the solar model. In the case of the neutrino flux, the discrepancy between models and observations (the "Solar Neutrino Problem") has not only remained, but has increased (Turck-Chièze et al. 1988; Sackmann, Boothroyd, & Fowler 1990). Proffitt (1994) including both helium and heavy element diffusion in his calculations, and using a different set of diffusion coefficients, has found an even larger increase in the expected neutrino fluxes.

The solar models presented here were constructed with the rotating version of the Yale stellar evolution code to which was added the treatment of thermal diffusion in the Bahcall & Loeb (1990, hereafter BL) and PM formulations. The constitutive physics differs from the Bahcall & Pinsonneault (1992a, b) models only in some details, such as the treatment of the Debye-Hückel corrections to the equation of state (see Guenther et al. 1992a). The models are fully described in Chaboyer (1993) and CDP. In § 2 we briefly describe the set of models calculated. In § 3 we compare the observed  $p$ -mode spectrum of the Sun to the standard solar model and the altered solar models that include a variety of combinations of the following physical processes: BL diffusion, PM diffusion, rotation, and the thin overshoot layer below the convection zone. The neutrino fluxes are discussed in § 4.

## 2. THE MODELS

The Sun is depleted in lithium by a factor of 100–200, and beryllium by 0.3 dex, with respect to the meteorites (Anders & Grevesse 1989). To help explain the observed lithium surface

abundances, it has been suggested that turbulent mixing takes place during evolution below the convection zone (Schatzman 1969, 1977; Vauclair et al. 1978; Vauclair 1988). Deliyannis (1990) has emphasized the importance of explaining both lithium and beryllium depletion simultaneously by the same mixing process. Such mixing is needed even when the effects of diffusion of light elements are taken into account. It has also been pointed out that the presence of a fully mixed layer at the base of the convection zone, due to convective overshoot or penetration, could be detected using solar seismology (Berthomieu et al. 1993; Monteiro, Christensen-Dalsgaard, & Thompson 1993; Stix 1994). In rotating solar models (Pinsonneault et al. 1989, 1990), the existence of a mixed layer follows naturally from the large rotational shear at the base of the convection zone which generates turbulence, and in turn mixes the shear layer. In this work we examine the significance of diffusion, rotation, and the mixed layer within the context of the  $p$ -mode oscillation spectrum. To carry out this investigation we have constructed a variety of models that include some combination of the effects of rotation, diffusion, and a mixed layer. In addition, because some authors model the overshoot layer by extending the adiabatic temperature gradient below the base of the convection zone we also calculate solar models with this formulation of overshoot.

Specifically we have constructed solar models with the following characteristics:

1. ssm—a standard solar model
2. nodh—a standard solar model without the Debye-Hückel correction to the equation of state
3. adov01—a standard solar model with a thin ( $0.1 H_p$ ) mixed adiabatic layer below the convection zone
4. adov03—a standard solar model with a thin ( $0.3 H_p$ ) mixed adiabatic layer below the convection zone
5. mx01—a standard solar model with a thin ( $0.1 H_p$ ) mixed radiative layer below the convection zone
6. pm—a standard solar model with helium diffusion in the PM formulation
7. pm08—a standard solar model with helium diffusion in the PM formulation but with the diffusion efficiency reduced by 20%
8. bl—a standard solar model with helium diffusion in the BL formulation
9. d+r—a standard solar model with rotation plus helium diffusion (PM formulation but with the diffusion efficiency reduced by 20%)
10. d+rZ—a standard solar model with rotation plus helium diffusion as in model d+r, but with the same  $Z$  as model pm08.

The standard solar model and its variations are nearly identical to the models described in CDP. Two modifications were made to accommodate the  $p$ -mode frequency calculations: (1) a detailed atmosphere based on the Krishna Swamy (1966) empirical fit to the solar atmosphere was added to the models, and (2) the inner most mass shell was positioned closer to the actual center (required for accurate  $l = 0$   $p$ -mode frequency calculations).

The models and some of their key distinguishing characteristics are summarized in Table 1. The mixing length and helium abundance of each model have been adjusted so that the radii and luminosities of the models match each other to one part in  $10^6$  ( $L_\odot = 3.8515 \times 10^{33}$  ergs  $s^{-1}$ ;  $R_\odot = 6.9598 \times 10^{10}$  cm) at the common age of 4.55 Gyr. The solar age from the zero-age

TABLE 1  
MODEL CHARACTERISTICS

Model	Description	Z	$Y_{\text{init}}$	$Y_{\text{surf}}$	$\alpha$	$\log T_{\text{core}}$	$\log \rho_{\text{core}}$	$x_{\text{b.c.}}$	$M_{\text{b.c.}}/M_{\odot}$	$T_{\text{b.c.}}$ ( $10^6$ K)	$\Delta\nu$ ( $\mu\text{Hz}$ )	$^{37}\text{Cl}$ (SNU)	$^{71}\text{Ga}$ (SNU)	$^9\text{B}$ ( $10^6$ ergs $\text{cm}^{-2}$ )
ssm	Standard solar model	0.01880	0.27077	0.27077	1.975	7.1910	2.1747	0.7188	0.020868	2.1246	136.98	6.69	124	4.66
nodh	No Debye-Hückel correction	0.01880	0.28167	0.28167	1.925	7.1927	2.1824	0.7249	0.019086	2.0740	137.01	7.20	127	5.06
adov01	0.10 $H_p$ overshoot adiabatic layer	0.01880	0.27051	0.27051	1.976	7.1912	2.1763	0.7216	0.021053	2.1430	137.01	6.73	125	4.69
adov03	0.30 $H_p$ overshoot adiabatic layer	0.01880	0.27064	0.27064	1.986	7.1912	2.1765	0.7209	0.021405	2.1510	137.05	6.74	125	4.70
mx01	0.10 $H_p$ overmixing layer	0.01880	0.27077	0.27077	1.976	7.1910	2.1747	0.7180	0.021041	2.1332	136.98	6.69	124	4.66
bl	Bahcall & Loeb diffusion	0.01956	0.27233	0.24610	2.091	7.1938	2.1812	0.7053	0.024737	2.2230	137.01	7.41	128	5.22
pm	Profitt & Michaud diffusion	0.01956	0.27208	0.24478	2.105	7.1943	2.1842	0.7025	0.025445	2.2259	137.02	7.55	129	5.33
pm08	pm with 80% diffusion efficiency	0.01941	0.27166	0.24674	2.080	7.1938	2.1826	0.7068	0.024857	2.2208	137.02	7.38	128	5.20
d+r	Rotation plus diffusion	0.01926	0.27122	0.25536	2.058	7.1927	2.1689	0.7081	0.023892	2.2215	137.05	7.15	127	5.03
d+rZ	d+r with Z of pm model	0.01941	0.27183	0.25600	2.059	7.1930	2.1686	0.7090	0.023762	2.2103		7.22	128	5.08



main sequence has been estimated by Guenther, Jaffe, & Demarque (1989, revised in 1992a) to be  $4.52 \pm 0.03$  Gyr. All the models except the adov01 and adov03 models were evolved from the fully convective pre-main-sequence phase. They reached the main sequence in about 0.04 Gyr.

The models constructed with a mixed adiabatic layer extending below the base of the convection zone (adov01 and adov03) were evolved from a ZAMS model (produced during the ssm evolutionary run). This introduces a small error of the order of one part in  $10^5$  in the temperature and density because of some loss of information in the entropy term. The effect on the  $p$ -mode spectrum is negligible. But it should be kept in mind that there is a small systematic difference in the calculated neutrino flux of  $+0.04$   $^{37}\text{Cl}$  SNU compared to evolving from the pre-main sequence. We did not evolve these models from the pre-main sequence because the numerical algorithm to extend the adiabatic temperature gradient below the surface convection zone lead to convergence problems as the model ceased being fully convective. This difficulty is only encountered when we impose the high tolerances (few parts in  $10^7$ ) necessary for solar  $p$ -mode calculations, and is not present if we relax the tolerances to standard stellar values. The question of how to properly treat the ad hoc inclusion of adiabatic overshoot during pre-main-sequence evolution will be investigated separately. Here we avoid the question all together by starting our evolution for these two models on the ZAMS.

### 3. THE $p$ -MODE FREQUENCIES

#### 3.1. Introduction

Diffusion affects the structure of the solar model in three ways: (1) it increases the helium abundance in the interior and decreases it in the convective envelope; (2) it increases the depth of the convective envelope (Guenther 1994) and (3) it changes the surface value of  $Z$  since  $Z/X$  is fixed by observation. Those  $p$ -modes whose inner turning points are near the predicted base of the convective envelope,  $l = 30$ – $50$  (see, e.g., Fig. 1 in Guenther 1994), will be the most sensitive to the effects that diffusion has on the position of the base of the convection zone. The frequencies of lower  $l$ -valued  $p$ -modes, which penetrate deeper will be affected by the increased interior helium abundance, and the higher  $l$ -valued  $p$ -modes, which probe only inside the convection zone, will be affected by the decreased helium abundance of the convective envelope.

The frequencies of  $p$ -modes with  $l$ -values less than 100 were calculated for the set of models. In Figure 1 we show the frequency difference between the models and the observations (Libbrecht, Woodard, & Kaufman 1990). The mx01 (the standard solar model with an overmixing layer extending  $0.1 H_p$  below the convection zone) is not shown because the model is identical in all respects to the ssm model. The d+rZ model is not shown as it was only constructed to test our understanding of the effects of rotation and metallicity on the neutrino fluxes. Lines connect  $p$ -modes with common  $l$ -values (not all  $l$ -values are plotted, only a representative sample). Bold lines are drawn for  $l = 30, 40,$  and  $50$ ; these modes are very sensitive to the conditions near the base of the convection zone. If the  $p$ -mode spectrum of a model were identical to the observed spectrum, then the plot would appear as a series of overlapping horizontal lines passing through  $v_{\text{model}} - v_{\odot} = 0 \mu\text{Hz}$ .

The agreement between theory and observation is not perfect. Errors in the modeling of the superadiabatic layers and the surface boundary conditions are responsible for the non-

horizontal slope of the lines (Guenther 1994). Errors in the surface layers affect the frequencies of all modes, primarily by altering the average spacing between adjacent in  $n$  modes. This is seen as a nonzero slope in the frequency difference diagram. In the model this error is associated with the difficulties of modeling turbulent convection, calculating low-temperature opacities that include the effects of molecules, and modeling the surface boundary condition. For the purposes of this work, we will ignore these problems and focus on errors that we can actually address.

Even when we ignore the nonzero slope of the lines, there still remains the error associated with the scatter of the lines, that is, the thickness of the bundle of lines. Modes with different  $l$ -values penetrate to different depths, hence the fact that lines joining different  $l$ -value modes do not lie on top of each other tells us that the interior model is not correct to varying degrees at different depths. Improvements to the modeling of the interior, via opacities, equation of state, inclusion of diffusion, etc., should reduce the spread or thickness of the bundle of iso- $l$   $p$ -mode frequency lines.

#### 3.2. Debye-Hückel Correction and the Significance of the Perturbations

A casual comparison of the thickness of the bundles of lines in the plots of Figure 1 reveal little difference among the models. The effects of diffusion, overshoot, and rotation are not that large, producing shifts in the frequencies of at most  $\pm 3 \mu\text{Hz}$ . The biggest difference appears between Figure 1a and 1b. Figure 1a corresponds to our standard solar model, that is, our reference model for this work. Figure 1b corresponds to a solar model identical to our standard solar model in all respects except that the Debye-Hückel correction has not been applied to the equation of state calculation. Improvements to the equation of state and the opacities have already been shown (Guenther et al. 1992a; Christensen-Dalsgaard, Däppen, & Lebreton 1988; Cox, Guzik, & Raby 1990; Christensen-Dalsgaard 1988; Ulrich & Rhodes 1983) to lead to improved  $p$ -mode frequency calculations. Here we use the model lacking the Debye-Hückel correction not only to emphasize its importance in producing accurate solar models for  $p$ -mode frequency determinations but also to provide a reference against which we will judge the significance of the effects of diffusion, overshooting, and rotation on the  $p$ -mode frequencies.

Clearly the effects of diffusion and rotation as indicated by the rest of the plots in Figure 1 are not as large as the Debye-Hückel improved equation of state (the nodh model is the only model that does not include the Debye-Hückel correction). Therefore, all conclusions that we draw regarding the beneficial or detrimental aspects of diffusion, and rotation, as implied by the  $p$ -mode oscillation spectrum, will not be as strongly supported as is the case for the Debye-Hückel correction.

#### 3.3. Overmixing and Convective Penetration

We have noted the importance of the existence of a thin mixed layer at the base of the convection zone, which is needed in the rotation models to describe the shear layer. Since there is some question about the effective mean temperature gradient in this layer, we have considered two extreme cases. In the first case, we simply have allowed the mixed layer to be chemically homogenized beyond the base of the convection zone by  $0.1 H_p$ , and have let the Schwarzschild criterion determine whether the local temperature gradient is radiative or convective

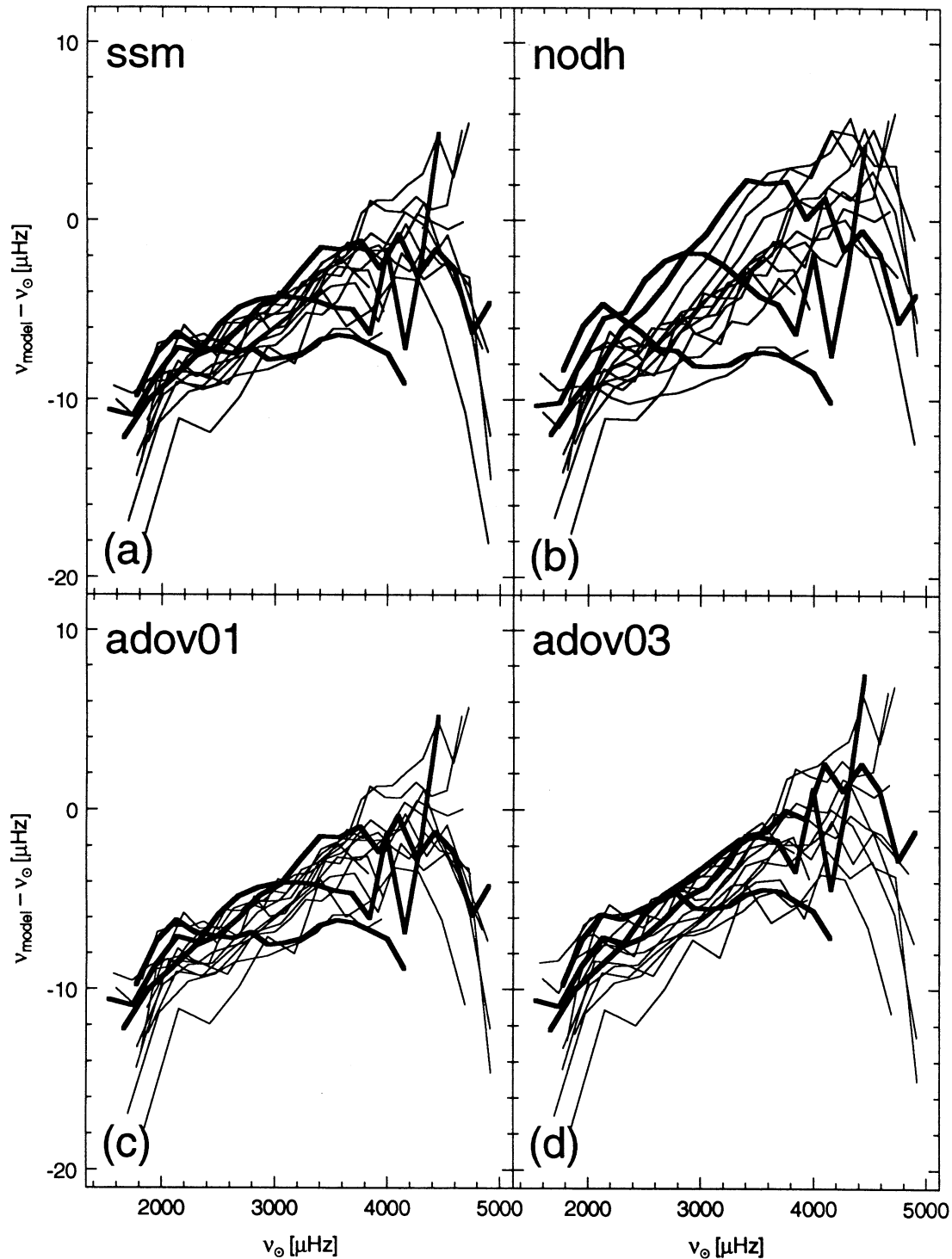


FIG. 1.—Frequency differences, model minus observed (Libbrecht et al. 1990), of a sample of  $p$ -modes are plotted opposite the observed frequency of the mode. Lines join common  $l$ -value modes for  $l = 0, 1, 2, 3, 4, 10, 20, 30, 40, 50, 60, 70, 80, 90,$  and  $100$ . The  $l = 30, 40,$  and  $50$  modes are connected by a thick line. Individual plots correspond to the different solar models as identified in the text and Table 1.

(overshooting in the terminology of Zahn 1991). Since the opacity is unaffected by the mixing in this case, the layer is radiative (model mx01). In the second case, we have extended the convection zone by applying the convective (adiabatic) temperature gradient by  $0.1$  and  $0.3 H_p$  beyond the formal boundary of the convection zone (models adov01 and adov03,

respectively) (see Fig. 2). This procedure is often called convective overshoot (Ahrens, Stix, & Thorn 1992) and is referred to as convective penetration by Zahn (1991, 1993).

The model with an artificially mixed layer below the base of the convection zone model mx01) is identical, within the numerical accuracy of the model, to the standard solar model,

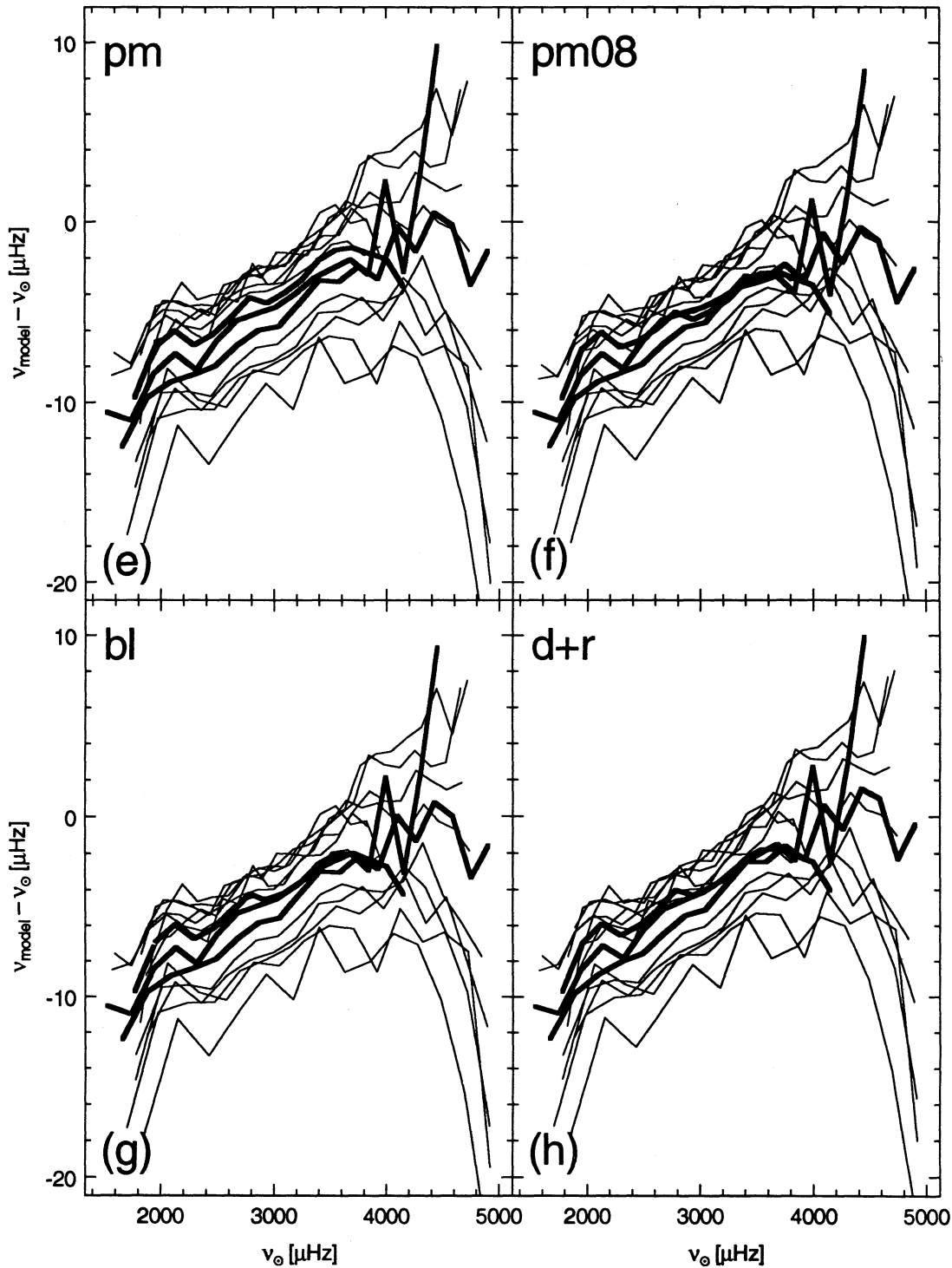


FIG. 1—Continued

and as a result the  $p$ -mode frequencies are also identical to standard model  $p$ -mode frequencies. This is expected because the region below the solar convection zone is chemically identical (except for trace amounts of Li and Be) to the convection zone. This is not the case in models that include diffusion, where, as a consequence, overmixing can lessen the penetration of He below the convection zone. We note that larger overshoot (by as much as  $0.3 H_p$ ) is required to match the solar Li

abundance in nonrotating nondiffused solar models (Ahrens et al. 1992) by enhanced pre-main-sequence burning. But overshoot above  $0.1 H_p$  seems ruled on other grounds, such as the observations of Li abundances in open star clusters which contain young solar analogs (Chaboyer et al. 1994b).

The  $p$ -mode frequencies of the solar model in which the adiabatic temperature gradient is extended  $0.1 H_p$  below the base of convection zone (adov01) are nearly identical to

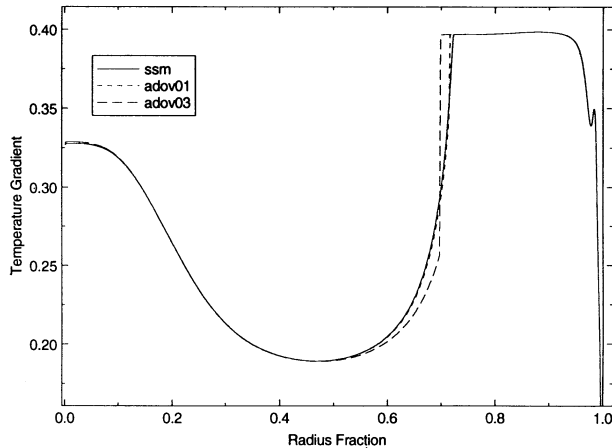


FIG. 2.—Temperature gradient as a function of radius fraction is shown for the ssm, adov01, and adov03 models.

the  $p$ -mode frequencies of the standard solar model. Not until the adiabatic overshoot reaches  $0.3 H_p$  do we see a significant affect on some of the  $p$ -modes. Comparing Figure 1d with 1a we see that only the  $l = 30$ – $50$   $p$ -modes are affected. The structure near the base of the convection zone (see Fig. 2) of the adov03 model is sufficiently different from the ssm model to perturb the frequencies of the  $p$ -modes that probe this region. We thus conclude that we are unable at this time, within the present uncertainties, to differentiate between radiative overmixing and convective penetration of the order of  $0.1 H_p$  in our models on the basis of the  $p$ -mode spectrum. Larger amounts of adiabatic overshoot (i.e.,  $\geq 0.3 H_p$ ) do reveal themselves in the  $p$ -mode spectrum but are ruled out by the constraints set by the observations of lithium destruction in the early phases of the evolution, of the Sun. Our conclusion differs from that reached by Berthomieu et al. (1993), who found an improvement in the  $p$ -mode spectrum of their convective penetration model over the standard solar model. Comparing their “delta frequency” plots with our plots (Fig. 1) reveals that the  $p$ -modes of their standard reference model are a factor of two more discrepant with observation than our standard reference model (ssm). We speculate that the substantial overshoot layer which they introduced may have compensated for the shortcomings of their reference solar model. This example illustrates the pitfalls in interpreting differences between theoretical models and observation when many unknown sources of error in the model remain.

### 3.4. Diffusion

Comparing Figures 1e (model pm), 1f (model pm08), and 1g (model bl) with Figure 1a (model ssm), it does not appear that diffusion improves the solar model at all. In fact, the spread of iso- $l$  lines is greater. Looking only at the  $l = 30, 40,$  and  $50$  lines (heavy stroked lines) we see that the scatter of these lines is reduced. Because the inner turning points of these modes are near the base of the convection zone, they are maximally sensitive to the position of the base of the convection zone—modes turning below the base are maximally perturbed by the abrupt change in the temperature gradient while those turning above are not. When the frequencies of these modes are compared to the frequencies of the Sun, an errant location in the base of the convection zone in the model is immediately discerned by the markedly greater spread of the  $p$ -mode frequency differences.

For a model with a poorly positioned convection zone base, the iso- $l$  lines for the  $l = 30$  to  $50$   $p$ -modes will be tightly bundled at low frequencies where all the  $p$ -modes have turning points above the base of the convection zone and then will fly apart ( $l = 30$  first, then  $l = 31$ , etc.) as the frequency increases and the turning points begin to dip below the base of the convection zone.

The location of the base of the convection zone in modes that include diffusion of helium, as noted by Guenther et al. (1993) and Guzik & Cox (1993), more closely agrees with the depth implied by inversion (Christensen-Dalsgaard et al. 1991), hence so do the  $l = 30$ – $50$   $p$ -modes. Unfortunately, not all the modes are improved. In fact, the overall scatter of the lines, that is, the bundle thickness for  $l = 0$  to  $100$ , is increased. There is still something wrong with this solar model. Guzik & Cox (1993) have succeeded in improving the agreement between model and observation beyond that represented here, by adjusting the opacities in their models. We believe that errors in the opacities just below the convection zone and in the outer layers and/or errors in the solar chemical abundances (within the present quoted uncertainties) may be responsible for most of the remaining discrepancy.

The two different formulations of diffusion tested here are insignificantly different, with regard to the  $p$ -mode frequencies. If Figures 1e, 1f, and 1g are superposed on top of each other then slight differences can be seen, but the magnitude of the differences are close to the magnitude of the numerical error associated with the model and pulsation calculation, which is approximately  $\pm 0.3 \mu\text{Hz}$  (Guenther et al. 1989). The pm08 model is identical to the pm model except the diffusion coefficients of the pm08 are scaled to 80% those in the pm model. We note that the frequencies of the  $l = 30$ – $50$   $p$ -modes of the pm08 model are nearly identical to the bl model and that the frequencies of the  $l = 30$ – $50$   $p$ -modes of the pm08 and bl models are more tightly bundled than the pm model. Because of the numerical uncertainties in the model calculation we do not attribute any significance to this.

### 3.5. Rotation plus Diffusion

The  $p$ -mode frequencies of the models that include rotation and diffusion (see Figure 1h; model d+r) are insignificantly different from the models that include just diffusion (compare Figs. 1e, 1f, and 1g). When compared to the standard solar model (Fig. 1a) the overall effect on the modes, like that of diffusion by itself, is to worsen the agreement. Despite the tremendous amount of effort required to produce the rotation plus diffusion models, we must conclude that rotation is not the missing ingredient that when included in the solar model calculation removes the remaining discrepancies between model and observation. Rotation hardly affects the internal structure and the  $p$ -mode frequencies at all.

However, rotation does tend to inhibit the diffusion, that is, a model with rotation and diffusion is similar to a pure diffusion model with approximately one-half the efficiency of diffusion (CDP). By intimately connecting the effects of rotation with diffusion (CDP) one perturbs the effects of diffusion on the solar model structure. The efficiency of diffusion is effectively reduced by the shear induced turbulence below the convection zone. Without rotation and the associated shear layer, helium diffuses out the base of the convection zone and remains within a diffusion scale length of the base ( $\sim 0.05 R_\odot$ ). With rotation, the shear induced turbulence below the convection zone spreads out the helium that has diffused out of the convection



zone into deeper layers, and it pushes some of the helium back into the convection zone, thereby reducing the effective efficiency of diffusion. Contrary to previous conclusions (Christensen-Dalsgaard et al. 1993; Guzik & Cox 1993), the presence of a (possibly rotationally induced) mixed layer due to turbulent diffusion just below the convection zone does not necessarily affect adversely the calculated  $p$ -mode spectrum. The effect on the  $p$ -modes depends only slightly on the adopted formulation of the diffusion coefficient.

### 3.6. Rotation Curve

While the approach of using the  $p$ -mode frequencies described in preceding sections provides a sensitive test of the effects of internal rotation on solar structure, it says little about the rotation curve itself. This is primarily because rotation is slow in these models and barely modifies their internal structure. But the distribution of angular momentum in the present Sun can be tested directly by observations of rotational splittings of oscillation modes which penetrate to different depths in the Sun, and with different amplitudes. Recent inversions of the lowest  $l$ -modes observed by the *IPHIR* space mission (Toutain & Frölich 1992), and of the IRIS network data (Loudagh et al. 1993), yield a rotation rate at  $0.2 R_{\odot}$  which is higher than the surface value and might be as high as 4 times the observed solar surface value, which is compatible with some of the models constructed by CPD. On the other hand, the inversion of intermediate  $l$   $p$ -mode splitting data (Libbrecht et al. 1990) yields a flat (solid body) rotation curve from the surface to  $r \approx 0.4 R_{\odot}$ . Even our minimal differential rotation model (see rotation curve in Fig. 3 and discussion below) predicts a rotation rate in the radiative envelope which is too high by at least a factor of two when compared to the rotation-rate-inferred  $p$ -mode splittings in the Sun. This conclusion is corroborated by a recent measurement of the solar oblateness (Sofia, Heaps, & Twigg 1994) which puts an independent limit on the amount of angular momentum in the outer parts of the solar radiative envelope which is  $2 \sigma$  below the oblateness of the minimal differential rotation model.

Another approach which has proved fruitful in testing models of the evolution of rotating stars is to appeal to observations of solar analogs at different stages of evolution. Observations

of the surface rotation periods of solar analogs more evolved than the Sun and now observed as subgiant stars are best explained if angular momentum buried deep in their interiors is being dredged up into their convective envelopes as their convection zones deepen (Pinsonneault et al. 1989; Demarque & Guenther 1988). There is in addition evidence from surface rotation velocities that evolved Sun-like stars have preserved a significant fraction of their initial angular momentum in their interiors into advanced phases of evolution (Pinsonneault et al. 1991). If these objects are true solar analogs, we must expect that there is similarly a reservoir of angular momentum in the deep interior of the Sun which is now buried in the inner 50% of its radius (or 90% of its mass). Some of this angular momentum will eventually be dredged up to the surface from the deep interior as the Sun evolves into a subgiant. Since all other evidence indicates that the outer radiative layers of the present Sun rotate slowly, this angular momentum must be located within the inner 50% of the solar radius. The rotation profiles of the models shown in Figure 3 exhibit too much rotation between 50% and 70% of the radius.

The minimal differential rotation model shown in Figure 3 was calculated assuming that the inhibitive effects of the mean molecular weight gradient can essentially be ignored and that the GSF rotational diffusion coefficients (James & Kahn 1970, 1971) are ten times larger than the default values used in the  $d+r$  model. The default values give the best fit to the observed rotation velocities, and Li abundances of young cluster stars. This minimal differential rotation model, which represents an extreme case, indicates that some important physical process or processes, which are responsible for removing some of the angular momentum from the outer radiative layers of the Sun, are missing in our formulation.

### 4. PREDICTED NEUTRINO FLUX

The structural effects of rotation are small, and so rotation by itself does not appreciably alter the predicted neutrino fluxes in solar models (cf. Bahcall 1989). However, recent studies have found that diffusion of  ${}^4\text{He}$  increases the predicted solar neutrino fluxes by approximately 11% for the Cl and water detectors, and 4% for the Ga detectors (Bahcall & Pinsonneault 1992a, b; Proffitt 1994). Rotational-induced mixing

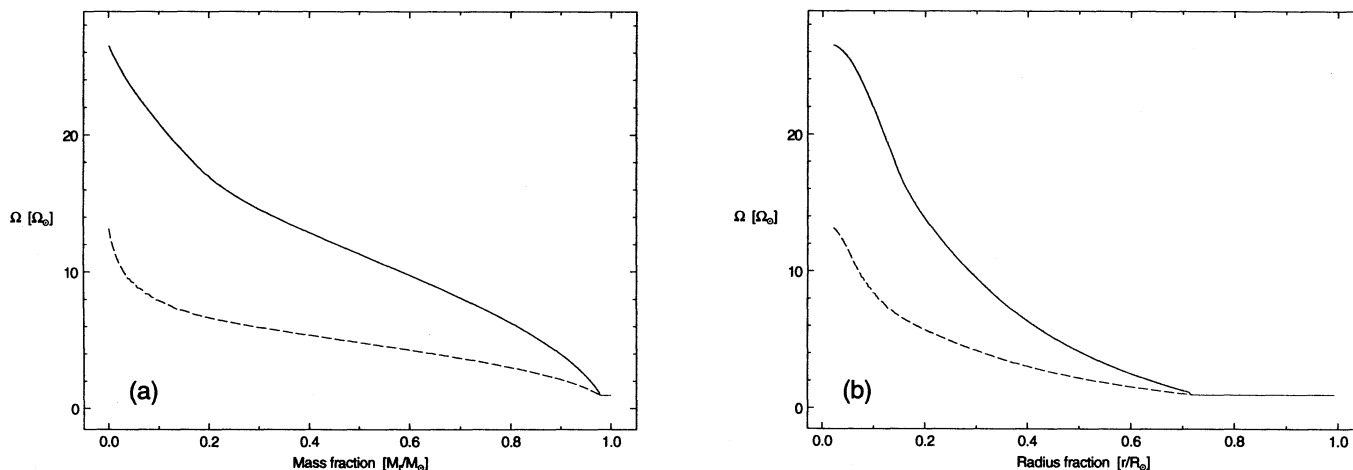


FIG. 3.—The angular rotation velocity is plotted as a function of interior mass fraction  $M_r/M_{\odot}$  (a) and radius fraction  $r/R_{\odot}$  (b) for the specific rotating model used in this paper ( $d+r$  or model VN of Chaboyer 1993), solid line, and for a minimal differential rotation model (model SF of Chaboyer 1993), dashed line.



counteracts the effects of diffusion, and so models which include the combined effects of rotation and diffusion predict neutrino fluxes between standard models (no diffusion), and pure diffusion models. This is shown in Table 1. We note that the neutrino flux of our ssm model is approximately 0.4 SNU for  $^{37}\text{Cl}$  lower than the best standard solar model of Bahcall & Pinsonneault (1992a). Our models use the nuclear energy generation of Bahcall (Bahcall & Pinsonneault 1992a), and the neutrino cross sections are also the same as those used in Bahcall & Pinsonneault (1992a). The difference in neutrino fluxes in the standard models is primarily due to the different final ages (4.55 Gyr vs. 4.6 Gyr of Bahcall & Pinsonneault), which results in a 0.2 SNU decrease, and a combination of the different final luminosities ( $3.8515 \times 10^{33}$  ergs  $\text{s}^{-1}$  vs.  $3.90 \times 10^{33}$  ergs  $\text{s}^{-1}$  of Bahcall & Pinsonneault) and different helium and heavy metal abundances (0.2708 and 0.01880 vs. 0.2716 and 0.01895 of Bahcall & Pinsonneault). There are also differences in the treatment of the Debye-Hückel correction (which here includes the effects of electron degeneracy), the opacity interpolation procedure, and in the chosen values of some of the physical constants.

The difference in solar neutrino fluxes predicted by our standard models and pure diffusion models,  $\sim 0.8$  SNU increase for  $^{37}\text{Cl}$ , are in good agreement with those found by Bahcall & Pinsonneault (1992a, b) and Proffitt (1994). The combined models, which include rotation and diffusion, predict solar neutrino fluxes which are between the standard models and the pure diffusion models, with rotation reducing the effect of diffusion on the neutrino fluxes by one-third. The last row in Table 1 (model d+rZ) summarizes the characteristics of a combined model in which  $Z$  has been adjusted to be the same as in the pure diffusion model pm08, with  $Z = 0.01941$ . We see that 0.18 SNU (or approximately half) of the reduction in the neutrino flux in d+r is due to the change in  $Z$ . The rest of the reduction (0.25 SNU) comes from the fact that the diffusion in the core is also being slowed down.

Including the effects of heavy-element diffusion does not appreciably affect the predicted  $p$ -mode frequencies, but has a comparable effect to  $^4\text{He}$  diffusion on the solar neutrino fluxes (Proffitt 1994). Proffitt (1994) determined neutrino fluxes of 9 SNU for Cl detectors, 137 SNU for Ga detectors, and  $6.48 \times 10^6$   $\text{cm}^2 \text{s}^{-1}$  for the  $^8\text{B}$  flux for models which include  $^4\text{He}$  and heavy-element diffusion, and suggested that turbulent mixing could reduce these amounts by a maximum of 0.17 SNU (Cl), 0.88 SNU (Ga) and  $0.13 \times 10^6$   $\text{cm}^2 \text{s}^{-1}$  ( $^8\text{B}$ ). Comparing no diffusion models, the pure diffusion model and the combined rotation plus diffusion model in Table 1 indicates that turbulent mixing due to rotation has a larger effect than that suggested by Proffitt (1994). The turbulent mixing induced by rotation will lead to reductions of 0.34 SNU (Cl), 1.8 SNU (Ga) and  $0.26 \times 10^6$   $\text{cm}^2 \text{s}^{-1}$  ( $^8\text{B}$ ) in the predicted solar neutrino fluxes of Proffitt (1994).

## 5. SUMMARY

### 5.1. Solar Internal Structure and Dynamics

We have considered the helioseismic properties of solar models which take into account rotationally induced mixing and its interaction with diffusion. We emphasize that rotating-diffusive models are very similar to structure to standard models of the Sun. But they differ from the standard models in several subtle but important ways (in particular the depth of the convection zone and the structure and composition profile

of the radiative layers just below the convection zone) which affects the  $p$ -mode frequencies.

We find that models that include diffusion (either the Bahcall-Loeb formulation or the Michaud-Proffitt formulation) and models that include diffusion and rotation predict  $p$ -mode frequencies that are in better agreement with observation for only those modes sensitive to the location of the base of the convection zone. Higher  $l$   $p$ -mode, confined to layers above the convection zone, are adversely affected. The effects on the  $p$ -mode frequencies of rotation cannot be distinguished from the effects of diffusion, although the latter produces a larger effect. With regard to rotation, the  $p$ -modes are only seeing the inhibitive effect that rotation has on diffusion via the shear induced turbulent layer, and the diffusion coefficients can be adjusted within their error to mimic the effect of rotation on the  $p$ -mode frequencies. At the present time, the errors associated with the calculation of the diffusion coefficients are too large to enable us to say one way or the other whether rotation is a necessary ingredient in the solar model. The diffusion coefficients can be adjusted within their error to mimic the effects of rotation on the  $p$ -mode frequencies.

On the other hand, as stellar astronomers we cannot ignore the considerable success of the rotating models in interpreting observations of other Sun-like stars of different ages (Pinsonneault et al. 1989, 1990; Chaboyer et al. 1994b). These results argue convincingly for the existence of a well-mixed transition layer in the Sun, most plausibly the result of rotational shear. From the  $p$ -mode frequencies, we are unable to distinguish between an adiabatic mixed layer (convective penetration) as favored by Zahn (1991, 1993), and a radiative mixed layer for overshoot layers less than  $0.1 H_p$ .

It is still very difficult to determine the rotation profile deeper inside the Sun. Here the observational errors are large, and the models more uncertain. There is however agreement over the main result that the inner part of the Sun rotates faster than its surface. This conclusion is in agreement with the preliminary inversions from the *IPHIR* space mission (Toutain & Frohlich 1992) and the IRIS network of whole-Sun observing stations (Loudagh et al. 1993). All observational evidence also points to slow rotation in the radiative layers just below the convection zone, which indicates that there is a missing angular momentum transfer mechanism in our models, which is very efficient in flattening out the solar rotation curve, at least in the outer part of the radiative envelope (see the rotation profiles in Fig. 3).

Even though the  $p$ -modes of our models do argue favorably for diffusion plus overshoot, the remaining errors in the model (and their affect on the  $p$ -mode frequencies) are large enough that if they cannot be accounted for as errors in the opacity and equation of state, then any combination of diffusion, overshoot, and overmixing, which is consistent with the observed surface abundances, is possible. We do not believe, though, this to be the case, and the simplest interpretation lies in opacity errors. We know that the opacities are accurate to only  $\pm 15\%$  and that smaller adjustments to the opacities can correct the  $p$ -mode spectrum of the solar model. Guzik & Cox (1993) have already shown that by tweaking the opacities, the  $p$ -mode frequencies of a solar model with diffusion can be made to agree with the observations to almost  $\pm 1$   $\mu\text{Hz}$ .

### 5.2. Solar Neutrinos

Because the rotating-diffusive models only differ in small and subtle ways from the standard solar model, the predicted

solar neutrino flux is not greatly affected by the presence of rotation and/or diffusion in the Sun. The results are summarized in Table 1. We see that the predicted neutrino flux is 6% lower in the rotation plus diffusion model compared to the diffusion only models.

Rotation by itself does not alter the predicted solar neutrino fluxes, as the structural effects of rotation are small. However, the mixing induced by rotation counteracts the effects of diffusion on the solar neutrino fluxes, such that our rotation-diffusion models have neutrino fluxes intermediate between the

Bahcall & Pinsonneault (1992a, b) standard and pure  ${}^4\text{He}$  diffusion models.

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