A Fuzzy Multiple Attribute Decision Making Model for Benefit-Cost Analysis with Qualitative and Quantitative Attributes

M. Ghazanfari and M. Mellatparast
Department of Industrial Engineering
Iran University of Science and Technology (IUST), Tehran, Iran

In this paper, a fuzzy multiple attribute decision making model to solve the mutually exclusive project selection problem with qualitative and quantitative attributes in benefit-cost analysis is proposed. The quantitative values such as investment cost, annual income and so on could be crisp or fuzzy numbers. Also, the qualitative attributes and their information are defined in fuzzy environment. By applying fuzzy set theory, we aggregate the qualitative and quantitative attributes with each other through some multiple attribute decision making models to select the best alternative for investment.

Keywords: Fuzzy Multiple Attribute Decision Making, Fuzzy Benefit - Cost Analysis, Fuzzy Ranking, Decision Theory, Linguistic Variables.

1. Introduction

One of the most effective techniques in evaluating investment projects is Benefit - Cost analysis. In traditional B/C technique, both benefit and cost values are supposed to be crisp numbers. However, due to the uncertainty and vagueness of information, it is often difficult to obtain the exact benefit and cost estimates such as investment cost, annual income, maintenance cost per year and so on. To deal with fuzzy data in Benefit - Cost analysis, Wang and Liang [1] have developed a fuzzy B/C analysis technique to deal with fuzzy numbers in project evaluation.

However, in some cases there are mutually exclusive projects in which some qualitative attributes as well as quantitative ones are to be taken into account. The traditional B/C analysis and other project evaluation techniques cannot deal with qualitative attributes. For example, suppose that you are going to construct a factory and three locations are feasible for that purpose. The related costs and benefits for these locations as well as some qualitative values are going to be considered for project evaluation. For instance, the term "weather conditions" is one of those qualitative attributes and the values it accept is linguistic terms such as "good", "very good" and so on. If we want to apply "weather conditions" as an attribute In our project evaluation, we see that it gives us fuzzy information in terms of a linguistic variable. Therefore, to apply qualitative attributes (like weather conditions) along with quantitative attributes (like investment cost, annual income, etc) in our project evaluation, we should develop a model to aggregate qualitative and quantitative attributes in order to have a better estimate for each project. Hence, a fuzzy multiple attribute decision making (FMADM) model has been constructed. No matter the quantitative attributes are fuzzy or crisp numbers this approach could be generally used in situations where you are supposed to integrate the qualitative and quantitative attributes.

2. Fuzzy Set Theory

Let X be a collection of objects, then a fuzzy set A in X is a set of ordered pairs
$A = \{(x, f_A(x) \mid x \in X\}$ where $f_A(x)$ is called the membership function of element x in set A which connects each element x in X to a real number in the interval [0,1]. X is called the universe.

2.1 Fuzzy Numbers

Fuzzy numbers are one of the most effective and common ways to represent fuzzy information. A fuzzy number "A" is a fuzzy subset of the real line R with membership function f_A which possesses the special properties[5].

Triangular and trapezoidal fuzzy numbers are two common fuzzy numbers which could easily represent the fuzzy information. The triangular fuzzy number can be represented by A= (a, b, c). A trapezoidal fuzzy number can be represented by A= (a, b, c, d). The "a" and "d" are the lower and upper bounds of assessment data and they used to represent the fuzziness of data. It should be noted that if b=c, the trapezoidal fuzzy number changes to a triangular fuzzy number. .

It should be noted that the tendency to use trapezoidal fuzzy numbers rather than triangular ones is related to their ease of use and compatibility to economical analysis[1]. For example, "approximately $ 500" can be represented by (495, 500, 505, 510). A non- fuzzy number "a" could be defined as (a, a, a, a).

2.2 Ranking Fuzzy Numbers

In engineering decision analysis, ranking alternatives under the existed criteria are essential. When we are dealing with crisp numbers, it is an easy task to obtain the best alternative. However, in a fuzzy Environment, it is usually difficult to say that a fuzzy number "A" is greater than a fuzzy number "B". For that reason, the ranking methods for fuzzy numbers has been developed [3]. Here, we use the technique developed by Wang and Liang [1], which is a method combining the methods proposed by Chen [3] and Kim & Park [6]. The technique could be simplified and formulated as follows:

Let $A_i = (c_i , a_i , b_i , d_i)$, i = 1, 2, ..., n , be n trapezodial fuzzy numbers. The ranking value of the trapezoidal fuzzy number A_i , $R(A_i)$, can be obtained by:

$R(A_i) =k [(d_i -x_1)/ (x_2 - x_1 - b_i + d_i)] + (1-k) [1-(x_2 - c_i) / (x_2 - x_1 + a_i - c_i)]$

where k is the index of optimism, $x_1 = \min [c_1 , ..., c_n]$ and $x_2 = \max [d_1 , ..., d_n]$

It should be noted that when you want to apply this ranking method, you should be aware of the behavior of the decision maker. According to the degree of risk acceptance, people can be classified into three groups: Risk seeking, Risk aversor and Neutral. When dealing with the first group, the amount of index k in the equation should be between 0.5 - 1. For a risk aversor person, the amount of k should be between 0 - 0.5. Finally, use k= 0.5 when you are dealing with a neutral person. Experience shows that people are usually risk aversor. Use of Utility Theory could be helpful to specify the decision maker's behavior.

3. Fuzzy Economic Measure in Fuzzy Benefit-Cost Analysis

Fuzzy engineering economics relations in Benefit- Cost analysis has been developed by Wang and Liang [1] . Here we introduce some of the related formulas for our purpose.

FEUAC : Fuzzy Equivalent Uniform Annual Cost

FCFAT : Fuzzy Cash Flow After Taxes

FMC: Fuzzy Maintenance Cost Per Year

FCRF: Fuzzy Capital Recovery Factor

FP: Fuzzy Initial Investment Cost

i: Fuzzy Investment Return Rate

n: Fuzzy Life Years of Project

$\tilde{1}$: non- fuzzy number “1” , *ie.* (1, 1, 1, 1)

The equations to obtain fuzzy measures in B/C analysis are as follows:

$$FCRF=(W_C , W_a , W_b , W_d)$$

$$W_C = [i_C (1+i_C)^{n_C}]/[(1+i_d)^{n_d} -1], W_a = [i_a (1+i_a)^{n_a}]/[(1+i_b)^{n_b} -1]$$

$$W_b = [i_b (1+i_b)^{n_b}]/[(1+i_a)^{n_a} -1], W_d = [i_d (1+i_d)^{n_b}]/[(1+i_C)^{n_C} -1]$$

4. Linguistic Variables

To represent too complex or too ill- defined situations in quantitative expressions, the concept of linguistic variable is very useful in dealing with vagueness and uncertainty existed in this kind of information. A linguistic variable is a variable whose values are words or sentence in natural or artificial language. For example, “intelligence” is a linguistic variable and its values could be “very low”, “low”, “medium” , “high”, “very high” and etc. We can also represent linguistic variables by using fuzzy set theory. For example, the linguistic variable “intelligence” and its corresponding values could be presented by a fuzzy set, A, A = {Very Low, Low, Medium, High, Very High}. The membership function of these values are shown in figure 2.

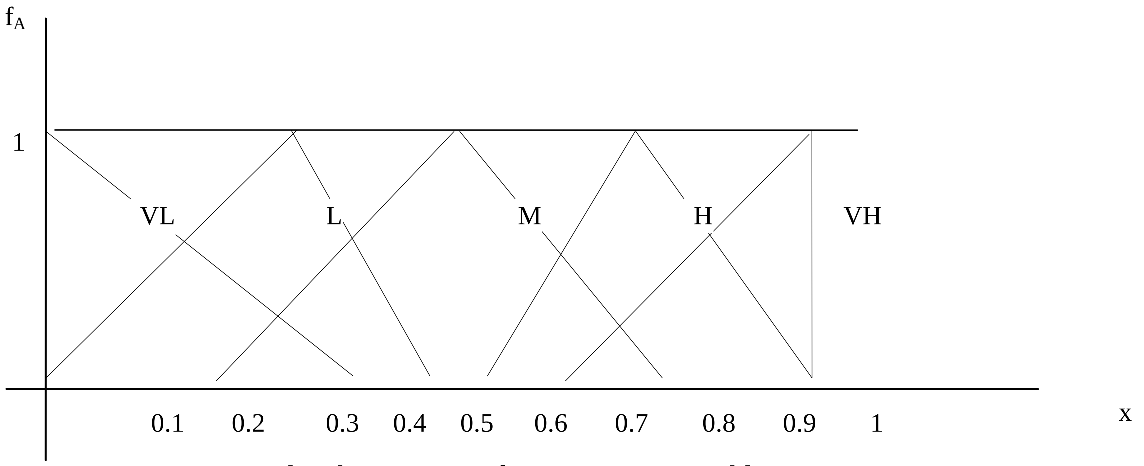

Figure 1. Membership Function for Linguistic Variable in Set A

We could also demonstrate linguistic variables by using fuzzy trapezoidal and triangular numbers. They could be represented as follows:

Very Low ≡ (0, 0, 0.3) High≡ (0.5, 0.7, 1)

Low = (0, 0.3, 0.5) Very High ≡ (0.7, 1, 1)

Medium = (0.2, 0.5, 0.8)

5. A Glance at MCDM [4]

Multiple criteria decision making (MCDM) refers to making decisions in the presence of multiple, usually conflicting criteria. In everyday life, many MCDM problems could be identified. For example, choosing a job based on its prestige, location, salary, promotion, working condition and so on.

MCDM problems could be divided into two categories ; Multiple objective decision making (MODM) and Multiple attribute decision making (MADM). The former tends to design the “best” alternative by considering the various interactions within the design constraints which best satisfy the decision maker by way of attaining same acceptable levels of a set of some quantifiable objectives whereas the latter relates to the selection of “an alternative” among a group of alternatives based on some attributes and their values. Where the MODM problems could be regarded as “design problem”, the MADM problems are identified as “selection problem”. Here, we introduce the MADM structure and some important techniques in this field.

5.1 Classification of MADM Models

For solving MADM problems based on attribute information processing, two approaches could be taken into account; Non- compensatory and compensatory models.

Non- compensatory models do not permit tradeoffs between attributes. The MADM methods which belong to this model are: Dominance, Maximum, Minimal, Cojuctive constraint method; Disjunctive constraint method and Lexicography method.

Compensatory models permit tradeoffs between attributes, *ie* in these models changes in one attribute can be offset by opposing changes in any other attributes. Compensatory models can be divided into three subgroups:

A) Scoring model: This model selects an alternative which has the highest score (or the maximum utility). Simple Additive Weighting (SAW), Hierarchical Additive Weighting and Interactive Simple Additive Weighting belong to this category.

B) Concordance model: This model arranges a set of preference ranking which best satisfies a given measure. Permutation, Linear Assignment and ELECTRE method belong to this model.

C) Compromising model: This model selects an alternative which is closest to the ideal solution. TOPSIS, LINMAP and Nonmetric MDS belong to this model.

5.2 Fuzzy Multiple Attribute Decision Making (FMADM) [3]

As stated in the earlier section, MADM methods are used to choose the best alternative among a group of alternatives. In classical MADM methods, it is assumed that all values are crisp numbers. However, in reality the value of each alternative for a given attribute could be crisp, fuzzy and/or linguistic. For example, suppose that three candidates are being considered for a professor position. The attributes used are creativity, maturity, communication skill and number of publications. The performance scores for the first three attributes are not quantifiable, rather they are represented by linguistic terms such as "good" , "average", "poor", etc. The forth attribute can be some integer numbers. This MADM contains a mixture of fuzzy and crisp number.

Fuzzy MADM methods are proposed to solve problems which involve fuzzy data. They are treated as a two-phase process. The first phase requires finding the fuzzy utilities (fuzzy final ratings) of alternatives. The second phase requires applying fuzzy ranking methods to determine the ranking order of alternatives.

Although FMADM methods have been derived from classical MADM methods and follow the MADM classification, their classification is based on four factors [3]. (1) Their capacity of solving large - size problems (2) The type of data allowed (3) The classical MADM method each fuzzy MADM relates to and (4) The technique each method uses. A more detailed definition of FMADM methods has been represented by Chen & Hwang [3].

6. Model Generation

To construct a model for fuzzy B/C analysis with qualitative attributes for mutually exclusive projects, we should pay attention to the fuzzy B/C analysis developed by Wang and Liang [1]. Here by using a flowchart we demonstrate the fuzzy B/C analysis technique for mutually exclusive projects.

(N : Number of available projects)

Figure 2. Fuzzy B/C Analysis Flowchart

6.1 The Proposed model

As it is clear, the traditional fuzzy B/C model developed by Wang & Liang [1] only deals with quantitative values, ***ie*** it is useful when it is impossible to assess the exact value of financial measures like annual cost, annual income, life time of project ,etc. To have an aggregated model for project evaluation based on quantitative and qualitative attributes, we should develop another model to consider them simultaneously. In the following, we define the quantitative and qualitative attributes in project evaluation. Furthermore, we explain the suggested approach to select the best project for investment among a group of mutually exclusive projects based on the attributes.

6.1.1. The need to apply qualitative attributes

The main question which may arise is "why do we need to apply qualitative attributes in Benefit - Cost analysis". Here we try to specify this argument in detail.

Broadly speaking, the investment projects can be classified into two groups: Private projects and Governmental (public) projects . In private projects, a private firm or an organization wants to evaluate the benefits and costs of projects based on his own goals or policies, *ie* for a private firm the objective of an investment project is to maximize the benefits and minimize the cost for the **overall firm**. However, in governmental projects, the government as the representative of the whole society is supposed to select projects which maximizes the benefits and minimizes the costs for the **overall society**. Therefore, the concept of cost or benefit for a governmental project may differ from that of a private project. Attributes such as air pollution, economical growth, inflation rate, and other social, cultural and political attributes are of great importance to government, but are less important to a private sector. For applying these attributes in project selection problem, we should define a model to accept the qualitative and quantitative attributes at the same time. The suggested procedure to deal with the problem has been explained in the following sections.

6.1.2 Suggested Procedure: A Fuzzy MADM Approach

In the previous section, we paid attention to the necessity of applying qualitative attributes in project selection problem mainly in governmental projects. In this case, we are facing with two types of attributes: Quantitative (annual cost, investment cost, etc.) and Qualitative (political, social and cultural) attributes. It is clear that the traditional fuzzy B/C algorithm [1] couldn't be used, since it only deals with quantitative attributes. Therefore, a new approach should be developed. We suggest FMADM approach and describe it as follows:

It should be noted that a project selection problem is basically a decision making problem rather than a project evaluation problem, *ie* it belongs to the family of decision making problems in which we decide which project is the most suitable project for investment. As we want to consider the qualitative and quantitative values in our evaluation, we are dealing with a multiple attribute decision making (MADM) problem. For quantitative attributes, we have an algorithm to integrate them to gain the equivalent measure of them, *ie* the B/C ratio shows the relationship between benefits and costs in each project [1], but for qualitative attributes, we can't integrate them because they have different characteristics. Since we are facing with a multiple criteria decision making problem, the MADM models are suitable methods. As the value of attributes are specified in fuzzy environment, the FMADM methods seem to be the best approach for project selection problem in which there is one quantitative attribute (B/C ratio) and some qualitative attributes. This approach has been demonstrated graphically in Figure 3.

To compare the similarities and differences between the proposed algorithm in this paper and the traditional fuzzy B/C analysis method, we pay attention to the steps to be taken in these two methods. The first two steps are similar in the two methods to assess the B/C

Figure 3. Fuzzy B/C Analysis with Qualitative Attributes Flowchart

ratio for the existed projects. However, the 3, 4 and 5 th steps are developed in this paper to obtain the qualitative information for the projects. It is the decision maker's responsibility to specify the effective qualitative attributes. In step 4, the information of qualitative attributes is gathered. These information are in terms of linguistic variables. In the 5 th step, we change the qualitative values to fuzzy numbers. Based on the information gathered in step 2 and step 5, we construct the final decision matrix. One column in the final decision matrix relates to the B/C ratio and the other columns relate to the qualitative attributes presented in step 3. According to the structure of final decision matrix in step 6, the best approach to select the best alternative for investment is to use fuzzy MADM methods. Therefore, in the 7 th step, the attribute weights should be given by the decision maker. Based on the information on the attributes and the tradeoffs between them, the best FMADM method which corresponds to the situation could be applied. Finally, as the final value of each alternative may be in terms of a fuzzy number, by applying the ranking method the best alternative for investment can be chosen.

6.2 Example[1]

To illustrate how the proposed method in this paper works, a hypothetical project selection problem is designed here to demonstrate the necessary steps should be taken to select the best alternative. Since the structure of the final decision matrix best fits to SAW method, we use this techniques to select the best alternative. However, Based on the tradeoffs between attributes, the selection of MADM methods may vary from one problem to the other.

Suppose that a government agency has three mutually exclusive projects A_1 , A_2, A_3. The estimated fuzzy investment cost, fuzzy maintenance cost per year and annual fuzzy cash flow after taxes (FCFAT) are given. The life time of each project is between 30 and 32 years, *i.e.* (29.5, 30, 32, 33) with no salvage value.

The interest rate is approximately 8% , *i.e* (0.078, 0.08, 0.08, 0.082).

Project	Initial Investment Cost ($\$ \times 10^6$)	Maintenance Cost Per Year ($\$ \times 10^6$)	Annual FCFAT ($\$ \times 10^6$)
1	approximately 10 million (9.9, 10, 10, 10.2)	approximately 0.035 million (0.031, 0.035, 0.035, 0.038)	approximately between 0.9 and 1 million (0.89, 0.9, 1, 1.02)
2	approximately between 12 and 13 million (11.9, 12, 13, 13.1)	approximately 0.038 million (0.037, 0.038, 0.038, 0.039)	approximately 1.1 million (1.05, 1.1, 1.1, 1.18)
3	approximately 15 million (14.9, 15, 15, 15.3)	approximately between 0.045 and 0.047 million (0.043, 0.045, 0.047, 0.048)	approximately 1 million (0.97,1, 1, 1.02)

Table 1. The fuzzy data of three government projects

Step 1

By using fuzzy B/C formulas [1], the annual costs and annual incomes can be obtained:

$FEUAC_i = FP_i \,.\, FCRF + FMC_i$

$FEUAC_1 = FP_1 \,.\, FCRF + FMC_1 = (0.6, 0.78, 1.07, 1.42)$

$FEUAC_2 = FP_2 \, . \, FCRF + FMC_2 = (0.72, 0.94, 1.38, 1.81)$
$FEUAC_3 = FP_3 \, . \, FCRF + FMC_3 = (0.9, 1.17, 1.6, 2.12)$

Step 2

The fuzzy measure of benefit (FCFAT) to cost (FEUAC) ratio of alternative A_1 , A_2 , A_3 are:

$FCFAT_1 / FEUAC_1 = (0.63, 0.84, 1.28, 1.7)$
$FCFAT_2 / FEUAC_2 = (0.58, 0.8, 1.17, 1.64)$
$FCFAT_3 / FEUAC_3 = (0.46, 0.63, 0.85, 1.13)$

Step 3

The effective qualitative attributes should be stated by the decision maker. Here, suppose that the decision maker specifies three attributes: Transportation availability, Availability of skilled workers and climatic conditions.

Step 4

A fuzzy set A = { Very Low, Low, Medium, High, Very High} is specified and the decision maker chooses the appropriate values that correspond to each alternative. The information has been presented in Table 2.

Project	Transportation Availability	Availability of skilled workers	Climatic condition
1	Medium	High	Very High
2	High	Very High	High
3	Medium	High	High

Table 2. The fuzzy information of qualitative attributes

Step 5

The linguistic variables in step 4 are to be changed to fuzzy numbers. The results are shown in Table 3.

Project	Transportation Availability	Availability of skilled workers	Climatic condition
1	(0.2, 0.5, 0.5, 0.8)	(0.5, 0.7, 0.7, 1)	(0.7, 1, 1, 1)
2	(0.5, 0.7, 0.7, 1)	(0.7, 1, 1, 1)	(0.5, 0.7, 0.7, 1)
3	(0.2, 0.5, 0.5, 0.8)	(0.5, 0.7, 0.7, 1)	(0.5, 0.7, 0.7, 1)

Table 3. The equivalent fuzzy number of qualitative attributes

Step 6

The final decision matrix which is the combination of qualitative and quantitative attributes could be demonstrated as follows:

Project	B/C	Transportation Availability	Availability of skilled workers	Climatic condition
1	(0.63, 0.84, 1.28, 1.7)	(0.2, 0.5, 0.5, 0.8)	(0.5, 0.7, 0.7, 1)	(0.7, 1, 1, 1)

2	(0.58, 0.8, 1.17, 1.64)	(0.5, 0.7, 0.7, 1)	(0.7, 1, 1, 1)	(0.5, 0.7, 0.7, 1)
3	(0.46, 0.63, 0.85, 1.13)	(0.2, 0.5, 0.5, 0.8)	(0.5, 0.7, 0.7, 1)	(0.5, 0.7, 0.7, 1)

Table 4. Final decision matrix of three mutually exclusive projects

Step 7

To apply the relative importance of each attribute, the decision maker specifies the following weights: W= (0.4, 0.3, 0.2, 0.1)

It shows that weights of B/C ratio, Transportation availability, Availability of skilled workers and climatic conditions are 0.4, 0.3, 0.2 and 0.1, respectively.

Step 8

Using Simple Additive Weighting (SAW) method, the fuzzy value of each alternative could be obtained:

$$U_i = \sum_{j=1}^{n} W_j X_{ij}$$

U_1 = (0.48, 0.73, 0.9, 1.22)

U_2 = (0.57, 0.8, 0.95, 1.26)

U_3= (0.39, 0.61, 0.7, 0.99)

Step 9

By applying Wang and Liang [1] ranking method taking k=0.5, the ranking values of each alternative are as follows:

R (A_1) = 0.5 R (A_2) = 0.56 R (A_3) = 0.36

As A_2 has the highest ranking value, it should be considered for investment.

It should be noted that the best alternative for investment based on Wang & Ling method [1] is A_1. By applying qualitative attributes in the proposed method in this paper, A_2 is the best alternative for investment.

8. Conclusion

In the project investment analysis there are situations in which the social, cultural and political outcomes are more important than the economical outcomes. Since the traditional engineering economics technique cannot solve the problem in which there are qualitative and quantitative attributes, by using fuzzy set theory and linguistic variables we're developed a model to deal with qualitative and quantitative attributes in Benefit-Cost analysis. Hence, a fuzzy multiple attribute decision making model is proposed to solve the problem.

Furthermore, developing a technique in the project evaluation in which there is a constraint in investment cost or situations where a combination of projects is allowed could be regarded as further research topic in this field.

References

[1] Liang, G. S. and M. J. J. Wang, (1995), "Benefit/ Cost Analysis Using Fuzzy Concept" , *The Engineering Economist*, SummerVol. 40, No. 4, pp. 359-376.

[2] Liang, G. S. and M. J. J. Wang, (1991), "A Fuzzy Multi-Criteria decision-making method for facility site selection", *The International Journal of Production Research*, Vol. 29, pp. 2313- 2330.
[3] Chen, S. J., and C. L. Hwang, (1992) *Fuzzy Multiple Attribute Decision Making*, Springer - Verlag,
[4] Hwang, C. L., and K. Yoon,(1981), *Multiple attribute Decision Making*, Springer - Verlag.
[5] Zimmerman , H. J., (1996), *Fuzzy Set Theory and Its Application*, Kluwer Academic Publisher, Third Edition.
[6] Kim K. and K. S. Park, (1990), "Ranking Fuzzy Numbers with Index of Optimism", *Fuzzy Sets and Systems*, Vol. 35, No. 5, , pp. 143-150.